

3/15/18

"The question isn't 'Who is going to let me?'; It's 'Who is going to stop me?'. " -Ayn Rand

HW: "Indefinite Integral" #1-6, 15-18  
Test 3 on Wednesday 3/28

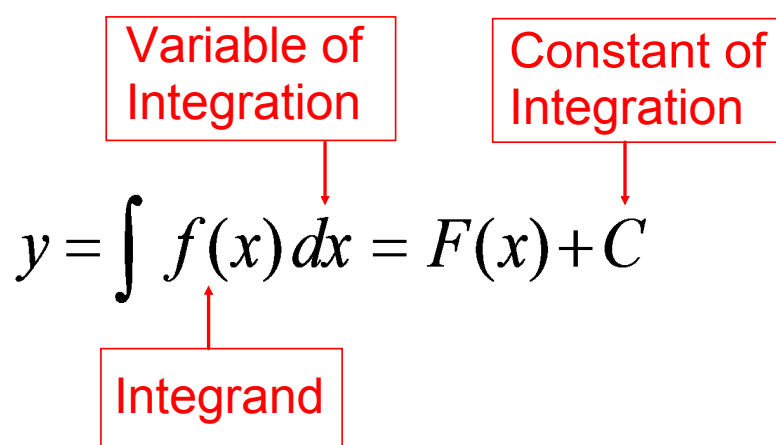
AIM: What is Integration?

## ANTIDERIVATIVES AND INDEFINITE INTEGRATION

### VOCABULARY AND DEFINITIONS:

1. A function  $F(x)$  is an antiderivative of a function if  $F'(x) = f(x)$  for all  $x$  in domain of  $f$ . The process of finding an antiderivative is antidifferentiation.
2. The family of all antiderivatives of a function  $f(x)$  is the indefinite integral of  $f$  with respect to  $x$  and is denoted by  $\int f(x) dx$
3. If  $F$  is any function such that  $F'(x) = f(x)$ , then  $\int f(x) dx = F(x) + C$  is called the general solution and  $C$  is called the constant of integration (an arbitrary constant).

General Solution is denoted by:



The diagram shows the general solution formula  $y = \int f(x) dx = F(x) + C$ . Three red boxes with arrows point to parts of the formula: 'Variable of Integration' points to  $dx$ , 'Integrand' points to  $f(x)$ , and 'Constant of Integration' points to  $C$ .

$$y = \int f(x) dx = F(x) + C$$

$\int f(x) dx$  read as the *antiderivative of  $f$  with respect to  $x$ .*

So, the differential  $dx$  serves to identify  $x$  as the variable of integration. The term indefinite integral is a synonym for antiderivative.

**EX #3:** Applying Basic Rules

$$\text{A.) } \int 3x' dx = \frac{3x^2}{2} + c$$

$$\text{B.) } \int 8 dx = 8x + c$$

$$\text{C.) } \int (x^2 - 3x + 4) dx = \frac{x^3}{3} - \frac{3x^2}{2} + 4x + c$$

**EX #4: Rewriting Before Integrating**

Original Integral	Rewrite	Integrate	Simplify
$\int \frac{1}{x^3} dx$	$\int x^{-3} dx$	$\frac{x^{-2}}{-2} + c$	$-\frac{1}{2x^2} + c$
$\int \sqrt{x} dx$	$\int x^{1/2} dx$	$\frac{x^{3/2}}{3/2} + c$	$\frac{2}{3} \sqrt{x^3} + c$
$\int 2 \sin x dx$	$2 \int \sin x dx$	$2(-\cos x) + c$	$-2 \cos x + c$

Recall:  
Derivative

$$3x^2$$



$$6x$$

$$3 \cdot x^2$$

$$3 \cdot 2x$$

$$6x$$

When integrating, think before you work! Practice will help you "discover" many tricks that make the integrations rules "fit your problems."

TRICK #1 REWRITE	TRICK #2 MULTIPLE or DISTRIBUTE	TRICK #3 SEPARATE FRACTIONS
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EX #5: Integrate the following:

A.)  $\int (3x-1)^2 dx$

$$\int (3x-1)(3x-1) dx$$

$$\int 9x^2 - 6x + 1 dx$$

$$= \frac{9x^3}{3} - \frac{6x^2}{2} + \frac{1x}{1} + C$$

$$= 3x^3 - 3x^2 + x + C$$

B.)  $\int \frac{x^2 + 4x + 5}{x^5} dx$

$$\int \frac{x^2}{x^5} + \frac{4x}{x^5} + \frac{5}{x^5} dx$$

$$\int x^{-3} + 4x^{-4} + 5x^{-5} = \frac{x^{-2}}{-2} + \frac{4x^{-3}}{-3} + \frac{5x^{-4}}{-4} + C$$

$$= \frac{1}{-2x^2} - \frac{4}{3x^3} - \frac{5}{4x^4} + C$$